ANALYSIS:

For neutral burning, the initial and final burning areas must be equal. Ideally, every burning area in between these two points would also be equal to both the initial and final burning areas. This is simply the definition of neutral burning.

Therefore, we shall begin by equating the formulae for the initial and final burning areas:

A_i = Burning areas of two end faces plus the burning area of the core
" $= (2)(\pi)(r_2^2 - r_1^2) + (2)(\pi)(r_1)(L_i)$
" $= 2\pi r_2^2 - 2\pi r_1^2 + 2\pi r_1 L_i$

A_f = Burning area at propellant outside diameter
" $= (2)(\pi)(r_2)(L_f)$
" $= 2\pi r_2 L_f$

But: $L_f = [\, L_i - (2)(r_2 - r_1)\,]$
So: $A_f = (2\pi r_2)[\, L_i - (2)(r_2 - r_1)\,]$
" $= (2\pi r_2)\,[\, L_i - 2r_2 - 2r_1]$
" $= 2\pi r_2 L_i - 4\pi r_2^2 + 4\pi r_1 r_2$

Equating the initial and final burning areas:

$$2\pi r_2^2 - 2\pi r_1^2 + 2\pi r_1 L_i = 2\pi r_2 L_i - 4\pi r_2^2 + 4\pi r_1 r_2$$

Next, rearranging terms and factoring, as needed, to solve for L_i gives:

$$2\pi r_1 L_i - 2\pi r_2 L_i = -4\pi r_2^2 + 4\pi r_1 r_2 - 2\pi r_2^2 + 2\pi r_1^2$$
$$L_i(2\pi)(r_1 - r_2) = -4\pi r_2^2 + 4\pi r_1 r_2 - 2\pi r_2^2 - 2\pi r_1^2$$
$$\text{"} \quad = -6\pi r_2^2 + 4\pi r_1 r_2 - 2\pi r_1^2$$

Multiplying both sides be -1 in order to change the signs of both sides:

$$L_i(2\pi)(r_2 - r_1) = 6\pi r_2^2 - 4\pi r_1 r_2 - 2\pi r_1^2$$

So: $$L_i = (3r_2^2 - 2r_1 r_2 - r_1^2)/(r_2 - r_1)$$

Algebraic division to see if numerator is evenly divisible by denominator:

$$\begin{array}{r|l}
 & \underline{3r_2 + r_1} \\
(r_2 - r_1) & 3r_2^2 - 2r_1 r_2 - r_1^2 \\
 & 3r_2^2 - 3r_1 r_2 \\
 & \underline{(-) \quad (+)} \\
 & 0 \quad r_1 r_2 - r_1^2 \\
 & \quad\; r_1 r_2 - r_1^2 \\
 & \quad\; \underline{(-) \quad (+)} \\
 & \quad\; 0 \;+\; 0
\end{array}$$

So, we see that the division gives an answer with no remainder, meaning that the expressions are, in fact, evenly divisible. Therefore, L_i is simplified to the following expression:

$$L_i = (3r_2 + r_1),$$

that is, the above equation shows the propellant length needed, for a given port radius and a given propellant external radius, to give an initial area equal to the final area. But, this does not ensure that the area at any time BETWEEN ignition and burnout will be constant and equal to both the initial and final areas. It is necessary to develop, analytically, the equation for the area at ANY time during burning, including initial and final areas. Then, the resulting equation must be evaluated to determine the TRUE variation (if any) of burning area and compare the result to the desired goal of constant burning area to see how well, or how poorly, they match.

At the instant that the propellant has burnt a distance, z, into the available web thickness $(r_2 - r_1)$, the area at that instant can by developed as follows:

$$\begin{aligned}
A_z &= (2)(\pi)(r_2^2 - (r_1 + z)^2) + (2)(\pi)(r_1 + z)(L_i - (2)(z)) \\
" &= 2\pi r_2^2 - 2\pi(r_1^2 + 2r_1z + z^2) + 2\pi(r_1L_i - 2r_1z + zL_i - 2z^2) \\
" &= 2\pi r_2^2 - 2\pi r_1^2 - 4\pi r_1z - 2\pi z^2 + 2\pi r_1L_i - 4\pi r_1z + 2\pi zL_i - 4\pi z^2 \\
" &= 2\pi r_2^2 - 2\pi r_1^2 - 8\pi r_1z - 6\pi z^2 + 2\pi r_1L_i + 2\pi zL_i
\end{aligned}$$

The distance, z*, that corresponds to the point at which the area may be a maximum (or minimum: remember, we still don't really know yet <u>which</u> it is!), assuming the area curve is non-neutral, is found by finding the value of z for which the slope of a line drawn tangent to the area curve is equal to zero, that is, a horizontal line. This area can be found by taking the derivative of the area equation with respect to z and setting the resulting derivative equal to zero, as follows:

$$\begin{aligned}
(dA_z/dz) &= d/dz\,[2\pi r_2^2 - 2\pi r_1^2 - 8\pi r_1z - 6\pi z^2 + 2\pi r_1L_i + 2\pi zL_i] \\
" &= 0 - 0 - 8\pi r_1 - 12\pi z + 2\pi L_i \\
" &= -8\pi r_1 - 12\pi z + 2\pi L_i \\
" &= (2\pi)(-4r_1 - 6z + L_i)
\end{aligned}$$

Setting this expression equal to zero gives:

$$\begin{aligned}
(2\pi)(-4r_1 - 6z + L_i) &= 0 \\
(-4r_1 - 6z + L_i) &= 0 \\
-6z &= 4r_1 - L_i
\end{aligned}$$

Multiplying both sides be -1 gives:

$$\begin{aligned}
6z &= L_i - 4r_1 \\
z &= (L_i - 4r_1)/6
\end{aligned}$$

But, we know that: $L_i = 3r_2 + r_1$

So:

$$\begin{aligned}
z &= ((3r_2 + r_1) - 4r_1)/6 \\
" &= (3r_2 + r_1 - 4r_1)/6 \\
" &= (3r_2 - 3r_1)/6 \\
" &= (r_2 - r_1)/2
\end{aligned}$$

Since the z at this location is called z*, then, in fact:

$$z^* = (r_2 - r_1)/2$$

This simply means that the maximum (or minimum) area occurs exactly one-half-way through the web thickness for ANY neutral-burning BATES grain design, no matter what its actual size!

We are not quite done, however, since we still don't know if the area becomes a maximum or a minimum at z^*. We need to know if the area curve is higher in the middle than at each end, or if it is lower than at each end. Mathematically speaking, a curve which is higher in the middle than at each end is said to be a "convex curve", the "legs" of which point "downward". A curve which is lower in the middle than at each end is said to be a "concave curve", the "legs" of which point "upward". The way in which this is determined is to take the derivative of the first derivative of the original equation, called the "second derivative", and evaluate its sign, that is, whether the result has a positive or negative sign. If the result has a positive sign, then the curve must be concave, with its legs pointing upward and the area at z^* must be a minimum area. If the sign is negative, then the curve must be convex, with its legs pointing downward and the area at z^* must be a maximum area.

$$(d/dz[dA_z/dz]) = d/dz(2\pi L_i - 12\pi z - 8\pi r_1)$$

$$" \quad = 0 - 12\pi - 0$$

$$" \quad = -12\pi$$

Remember, we aren't concerned with the actual result itself, merely its sign. As can be seen, the sign is negative. We now know that the area curve is a convex curve with its legs pointing downward, which means, in turn, that the area A_{z*} must be a maximum! So, the area at the midpoint of the burning phase is greater than the values at the initial and final points of burning. This is why it was stated earlier that a so-called neutral-burning BATES grain cannot ever be truly neutral: its geometry doesn't allow it.

It was previously mentioned that, even though the grain is not really neutral, it is still possible to come within a few percent of neutrality by the proper selection of dimensions. Therefore, it would be desirable to have some sort of equation that tells us the amount that the maximum area differs from the initial (and final) burning area. This would allow the designer to pick and choose the dimensions with full knowledge of the effects upon the area variation during burning.

$$A_z = 2\pi r_2^2 - 2\pi r_1^2 - 8\pi r_1 z - 6\pi z^2 + 2\pi r_1 L_i + 2\pi z L_i$$

But: $z^* = (r_1 - r_2)/2$

So: $A_{z*} = 2\pi r_2^2 - 2\pi r_1^2 - 8\pi r_1(r_2 - r_1)/2 - 6\pi((r_2 - r_1)/2)^2 + 2\pi r_1 L_i + 2\pi L_i(r_2 - r_1)/2$

Note: $((r_2 - r_1)/2)^2 = (r_2^2 - 2r_1 r_2 + r_1^2)/4$

So: $A_{z*} = 2\pi r_2^2 - 2\pi r_1^2 - 4\pi r_1 r_2 + 4\pi r_1^2 - (3/2)\pi r_2^2 + 3\pi r_1 r_2 - (3/2)\pi r_1^2 + 2\pi r_1 L_i + \pi r_2 L_i - \pi r_1 L_i$

Collecting terms:

$$A_{z*} = (1/2)\pi r_2^2 + (1/2)\pi r_1^2 - \pi r_1 r_2 + \pi r_1 L_i + \pi r_2 L_i$$

$$\text{“} \quad = \pi(r_2^2/2 + r_1^2/2 - r_1 r_2 + r_1 L_i + r_2 L_i)$$

Since its a simpler equation, let's use the equation for A_f as a comparison to the just-developed equation (the results should be exactly the same if the equation for A_i is used; it would just take longer!). We want to take the ratio of A_{z*} to A_f in order to find out the relationship between the two.

$$(A_{z*}/A_f) = [\,(\pi)(r_2^2/2 + r_1^2/2 - r_1 r_2 + r_1 L_i + r_2 L_i)] \,/\, [(2\pi)(r_2^2 - r_1^2 + r_1 L_i)]$$

$$\text{“} \quad = [(r_2^2/2 + r_1^2/2 - r_1 r_2 + r_1 L_i + r_2 L_i)] \,/\, [(2)(r_2^2 - r_1^2 + r_1 L_i)]$$

$$\text{“} \quad = [(2)(r_2^2/4 + r_1^2/4 - r_1 r_2/2 + r_1 L_i/2 + r_2 L_i/2)] \,/\, [(2)(r_2^2 - r_1^2 + r_1 L_i)]$$

$$\text{“} \quad = (r_2^2/4 + r_1^2/4 - r_1 r_2/2 + r_1 L_i/2 + r_2 L_i/2) \,/\, (r_2^2 - r_1^2 + r_1 L_i)$$

Remember that: $L_i = (3r_2 + r_1)$.

So:

$$(A_{z*}/A_f) = (r_2^2/4 + r_1^2/4 - r_1 r_2/2 + (r_1)(3r_2 + r_1)/2 + (r_2)(3r_2 + r_1)/2) \,/\, (r_2^2 - r_1^2 + (r_1)(3r_2 + r_1))$$

$$\text{“} \quad = (r_2^2/4 + r_1^2/4 - r_1 r_2/2 + (3/2)r_1 r_2 + r_1^2/2 + (3/2)r_2^2 + r_1 r_2/2) \,/\, (r_2^2 - r_1^2 + 3r_1 r_2 + r_1^2)$$

$$\text{“} \quad = ((7/4)(r_2^2) + (3/4)(r_1^2) + (3/2)(r_1 r_2)) \,/\, (r_2^2 + 3r_1 r_2)$$

Note that we have three unknowns with which we have to deal. It is a fact of mathematics that, in order to solve an equation or set of equations, there must be at least as many equations as there are unknowns. In this case, however, we seem to have three pretty much independent variables: L_i, r_1 and r_2. It would be nice to be able to manipulate something to reduce the unknowns to two. So, let's define a factor, k, which will be made equal to the ratio of the port radius, r_1, to the maximum propellant radius, r_2. That is: $k = (r_1/r_2)$ and, therefore, $r_1 = (k)(r_2)$. This allows us to specify the port radius in terms of the propellant radius. It also means that k can only range from greater than zero to 1, i.e., $0 < k <= 1$. Substituting this relationship into the above equation gives:

$$(A_{z*}/A_f) = ((7/4)(r_2^2) + (3/4)(k^2 r_2^2) + (3/2)(k r_2^2) \,/\, (r_2^2 + 3k r_2^2)$$

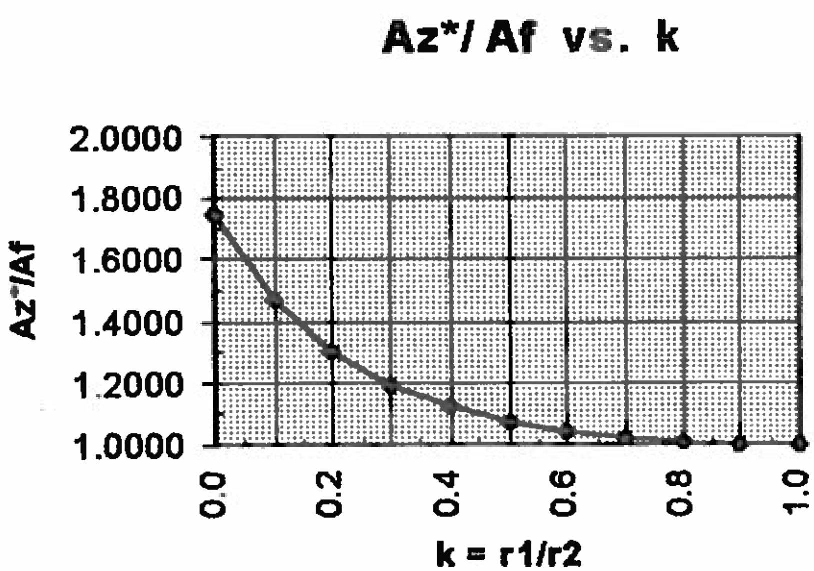
$$\text{“} \quad = [(r_2^2/4)(7 + 3k^2 + 6k)] \,/\, [(r_2^2)(1 + 3k)]$$

$$\text{“} \quad = (1/4)[(3k^2 + 6k + 7) \,/\, (1 + 3k)]$$

Testing reveals that this expression doesn't seem to be able to be reduced by means of division, such as was done previously (the test division produced an unwieldy remainder which was more difficult to work with than the equation above.)

Therefore, this is the final equation with which we must work. We know that k must not be less than or equal to zero (or we get into a physically impossible situation), nor greater than 1. We can substitute various values for k in this range into the equation and determine the ratio (A_{z*}/A_f). We can also graph the results in order to get a visual idea of what is going on. Also, by assigning a desired value to the ratio (A_{z*}/A_f) and using trial and error to plug in various values of k, we could find a value of k that gives the desired ratio. But, obviously, that would be a hard way to do it! It's easier to calculate several ratios for several values of k and then graph them and estimate what k value will give us the desired ratio. Although we know that a value of k = zero is not physically realizable (there would be no core!), it is possible to plug it into the equation as an extreme boundary condition just to see what we get. Further, even though a value of k = 1 means that there is no propellant(!), again we can plug it in as an extreme boundary condition . So, let's have some fun (if you feel uncomfortable plugging in these extreme limits, then just use a very, very small core diameter, for instance, 0.0001 inch and/or a value for r_2 which is almost, but not quite, equal to the maximum value of r_2, say, (r_2 - 0.0001 inches), the results should be nearly the same!)

k	(A_{z*}/A_f)
0.0	1.7500
0.1	1.4673
0.2	1.3000
0.3	1.1934
0.4	1.1227
0.5	1.0750
0.6	1.0429
0.7	1.0218
0.8	1.0088
0.9	1.0020
1.0	1.0000



Based upon the given data, and also by referring to the graph, we see that if we want to have the maximum area within about 5% of the initial or final areas, then we need a value for k of about 0.6. That is, the port radius should be about 0.6 x the propellant's maximum radius. Note something else here: although the maximum area would be within 5% of the initial or final area, it would be within about 2.5% of the average of the initial and final areas. This might very well be good enough for most experimental work.

Since we can pretty freely assign the dimensions we want, it is important to know that other factors impinge upon the selection of grain dimensions. For example, in order to ensure that there is little or no erosion of the grain during burning, the port area must be at least two times the throat area. It would be better if it were three or four times the throat area. So, if you have a half-inch throat diameter, a port diameter of at least D_t x the square root of 2, or about 0.707 inch is needed. Now, the rest of the grain dimensions will be determined by this port diameter, using the equations previously given. On the other hand, if the maximum engine casing and cartridge diameters are known, then the port diameter will be determined based upon the acceptable percentage area change desired. This will, in turn, dictate the throat diameter in order to avoid erosive burning. Or, perhaps a given burning time is desired and the burning rate at the desired operating pressure is known. Then, the web thickness is determined. It then has to be decided what all the other dimensions will be. Finally, throat area is also a function of how many cartridges are to be used. But, if too many cartridges are used, with the idea that the throat diameter can just be increased as needed to accommodate the additional thrust, then, eventually, the throat diameter will get large enough to start violating the erosive burning rule. It is obvious that engine/propellant design is composed of a series of tradeoffs, usually requiring a series of iterations of design until a satisfactory result is achieved.

EXAMPLE:

Given an engine diameter of 2.5 inches, and a wall thickness of 0.125 inch, and a cartridge wall thickness of 0.065 inch, find the dimensions of the propellant grain for near-neutral burning having about 5% area increase over the initial or final burning areas:

> The propellant diameter is: 2.5 " - 2 x 0.125 " - 2 x 0.065 " = 2.120 ", which makes the propellant radius 2.120 / 2 = 1.060 ".
>
> Consulting the table, for a k value of about 0.6, the area increase is just slightly less than 5%, which is close enough to what we need. So, the port radius is then 1.060 x 0.6 = 0.636 ". Then, the propellant length must be: 3 x 1.060 + 0.636 = 3.816 ". The initial surface area x 1. 0429 = the maximum burning area = $[(2)(\pi)(2.12^2 - 1.272^2) / 4 + (\pi)(1.272)(3.816)] \times 1.0429 = 20.6154$ sq. inches per cartridge.
>
> The nozzle throat area should be no more than one-half the area of the port, or, conversely, the port area should be at least twice the area of the nozzle throat. Since the ratio of the diameters of two different circles is proportional to the square root of their respective areas, then the diameter of the throat should be equal to the square root of the port-to-throat area ratio, that is the square root of two, in this example. So, the throat diameter should be no larger than (2 x 0.636) / 1.41414..... = 0.89944 ".
>
> The throat area is then: $(\pi)(0.89944)^2 / 4 = 0.63538$ sq. in. If the pressure is to be 500 psia, and the coefficient of force is about 1.4, then the thrust will be: 1.4 x 500 x 0.63538 = 444.8 lb. The solid propellant thrust equation is: F = A x Isp x

density x burning rate. Assume that Isp = 232 seconds, density = 0.056 lb./ cubic inch, burning rate = 0.20 inches/second, and the thrust is 444.8 lb. Then, the burning area needed is:

$$444.8 / (232 \times 0.056 \times 0.20) = 171.18 \text{ sq. inches.}$$

So, the number of cartridges needed is: 171.18 / 20.6154 = 8.3036.

This would be rounded off to 8 cartridges.

This concludes the development of the design of a near-neutral-burning BATES grain. I hope it will be of some use to those of you who would like to "strike out" on your own. At least, it should provide an understanding of the design process and show that it is not very mysterious, at all. Good luck, and feel free to write or e-mail any comments you have, positive or negative. My e-mail address is as follows:

larry79054@aol.com

Safety and the Burning Rate Exponent

or, Why Our Motors Don't Explode

by Robert Dahlquist

It is essential for anyone designing and building his own rocket motor to first research the burning rate characteristics of the propellant he is considering, then carefully consider the implications of those characteristics.

One of those characteristics is the temperature constant. An empirical constant in the burning-rate equation, it is the basic measure of how fast the propellant burns. It is a basic characteristic of the propellant type and formula. It increases somewhat if the propellant is warm or hot prior to firing. Conversely, it decreases slightly if the propellant is cold prior to firing. Hence the name, temperature constant.

This paper concerns itself with the other burning-rate characteristic, the pressure exponent. It is a measure of the propellant's sensitivity to pressure. A propellant with high sensitivity to pressure will have a high pressure exponent; a propellant with low sensitivity to pressure will have a low pressure exponent.

These characteristics can be measured by testing some of the propellant in a special chamber called a strand burner. Or, since the commonly used types of propellants have already been tested extensively, the literature available from various sources can be searched, and will reveal a range of values of these parameters for commonly used propellant types (and even uncommon ones that someone has already tested).

Rocket propellant burns faster under pressure. It needs some pressure in order to burn at the proper rate. The higher the pressure, the faster it burns. And the faster it burns, the higher the pressure it creates inside the motor. And the more pressure, the faster it burns, and so on.

Why doesn't this create a vicious cycle of ever-increasing, runaway pressure, ending only with explosion of the motor? With the wrong propellant, it could. But with the right kind of propellant, the rate at which gases can escape through the nozzle increases more rapidly with pressure than the burning rate.

On ignition, the burning propellant produces gases faster than they can escape through the nozzle, causing pressure to build up. The chamber pressure increases until the mass flow through the nozzle catches up with the mass flow produced by the propellant's burning rate. At that point, the chamber pressure stabilizes.

The rate at which the burning rate changes with pressure is called the burning rate exponent or pressure exponent, n. The burning rate in terms of mass flow of combustion products is given by:

$$m = A_b * \rho_b * a * P_c^n \qquad \text{(Equation 1)}$$

where:
- m = mass flow rate of combustion products, Kg/sec.
- A_b = burning surface area, m^2
- ρ_b = propellant density, Kg/m^3 of solid
- a = empirical temperature constant for the propellant
- Pc = chamber pressure, N/m^2 (Pascals) absolute
- n = pressure exponent of burning rate, a measure of the propellant's sensitivity to pressure

When m is plotted on log-log graph paper, it appears as a straight line with a slope equal to n. (In reality n does not remain strictly constant at all pressures, and a plot of real data makes a slightly curved or approximately straight line.)

The rate of mass flow through the nozzle throat is given by:

$$\dot{m} = A_t P_1 \sqrt{k\left(\frac{M}{R' T_1}\right)\left(\frac{2}{k+1}\right)^{(k+1)/(k-1)}}$$

(Equation 2)

Where: m = mass flow rate of combustion products, Kg/sec.

At = nozzle throat area, m^2

P_1 = chamber pressure at nozzle entrance, Pascals

k = ratio of specific heats, Cp/Cv

R' = universal gas constant, 8.3143 J/°K-mol

M = average molecular weight of combustion products, Kg/mol.

T_1 = nozzle inlet temperature of combustion products, °K

For the purpose of this discussion, we will lump together everything inside the square root sign and consider it a constant; although in reality there may be some change in M and k due to Le Chatelier's Principle. T_1 may also change somewhat; the changes in M and T_1 will be in the same direction, thus will cancel each other out to some degree. To simplify things we will also ignore, temporarily, the fact that the chamber pressure is not quite the same at both ends of the motor. Thus:

m = At *Pc.* constant (Equation 3)

Of course, the nozzle area is also constant, leaving Pc as the only variable. Thus, mass flow is directly proportional to the chamber pressure (in other words, has an exponent of 1.0), and when plotted on graph paper, it, appears as a straight line with a slope of 1.0.

The relationship between burning rate and chamber pressure can be visualized by plotting the nozzle flow (equation 2) and the burning rate (equation 1) together on the same graph (Figure 1). The operating point, or chamber pressure and mass flow rate at which the motor will operate, is given by the intersection of the two curves. At this point, gases are flowing out through the nozzle at the same rate they are being produced by the burning propellant.

Figure 1 shows the final stage of the ignition process. At time T_1, the propellant has just become fully ignited. The chamber has been partially pressurized by the igniter. At T_1, the burning propellant is generating hot gases faster than they can flow out through the nozzle, causing pressure to build up. This is shown on the graph as a higher value of mass flow for the propellant than for the nozzle.

At T_2, the mass flow through the nozzle has almost caught up with the burning rate. The rate of pressure rise is therefore tapering off. This appears as a rounding of the corner on a graph of chamber pressure (Pc) vs. time. (Figure 2)

At T_3, the mass flows are balanced; gases are being generated in the chamber at the same rate at which they are flowing out the nozzle. Thus, no further increase in pressure occurs, so long as the burning surface area remains constant.

What happens if the burning area (A_b) changes? Obviously, if the burning area increases, the pressure will go up; and if the burning area decreases, the pressure will go down. But how much? It depends on the pressure exponent. Figures 3 through 6 illustrate what will happen.

[**The following three paragraphs refer to Figure 2.** T_1, T_2, and T_3 are the same as in Figure 1. The solid line shows the idealized chamber pressure (Pc) vs. time, as measured at the nozzle inlet.

Pc at the forward bulkhead will be higher, due to the pressure drop (ΔP) caused by friction and turbulence of the gases flowing at high velocity through the grain port. This ΔP decreases as burning progresses, due to the increase in the port diameter.

The dashed line shows Pc at the forward bulkhead.

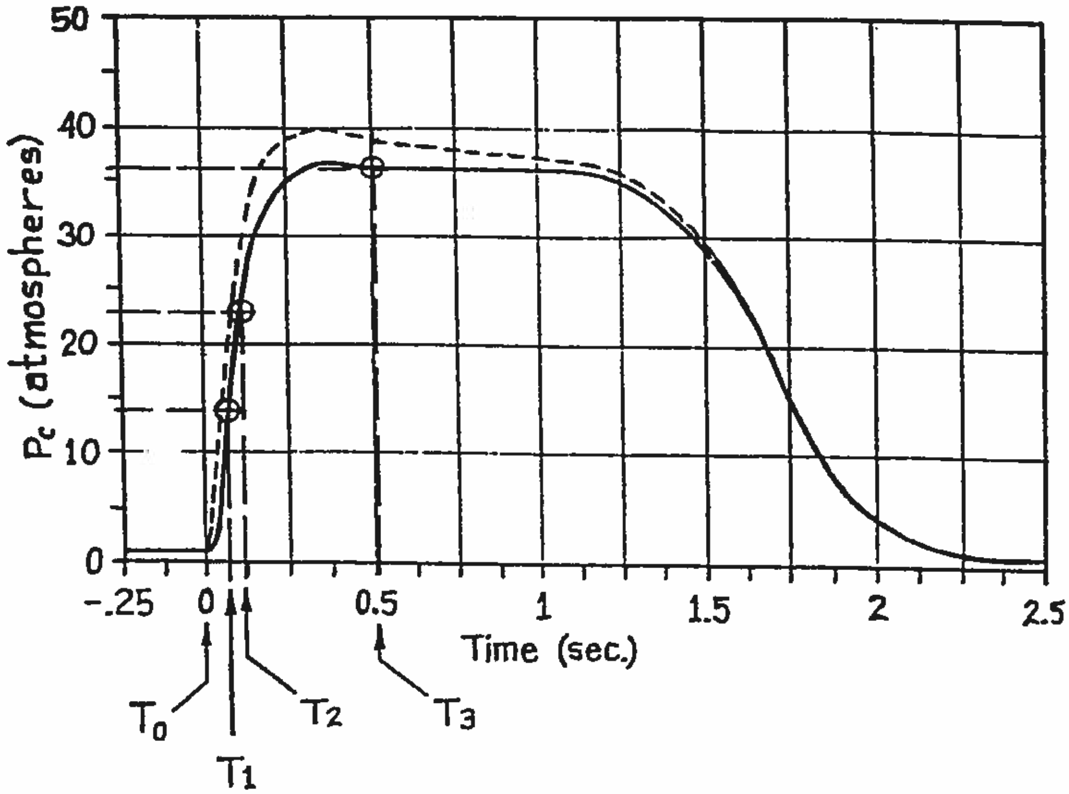


Figure 2 - Pressure Build-up

The actual peak Pc at this location depends on the grain design and the igniter's burning rate.]

Referring to Figures 3 through 6, each graph shows three different burning rates representing three different burning surface areas: nominal burning area, 30% over nominal area, and 25% under nominal area. This was done to illustrate what happens when the motor deviates from nominal, with different burning-rate exponents (values of n). Notice how the operating pressure changes with the change in burning area, and how the amount of change in Pc varies with the different values of n.

Other things will affect the mass flow in the same way as a change in burning area; initial temperature of the propellant before firing, for example. So the three burning rate lines on each graph could also be looked at as representing what happens when the propellant is hotter or colder before firing. If the propellant is hotter before firing, the burning rate curve shifts upward. If the propellant is colder, the burning rate curve shifts downward. Its slope remains the same.

What happens if the nozzle area changes? If the nozzle area decreases due to slag buildup, or a temporary blockage by any igniter parts, or propellant fragments, the nozzle mass flow curve will shift downward; its slope will remain the same. This is not shown on the graph, but you can take a ruler and draw in a line parallel to the existing line if you want to see what happens. If the nozzle area increases due to erosion, the line shifts upward and its slope remains the same.

As the exponent n increases, the change in pressure becomes much greater in proportion to the change in burning surface. As n approaches 1.0,

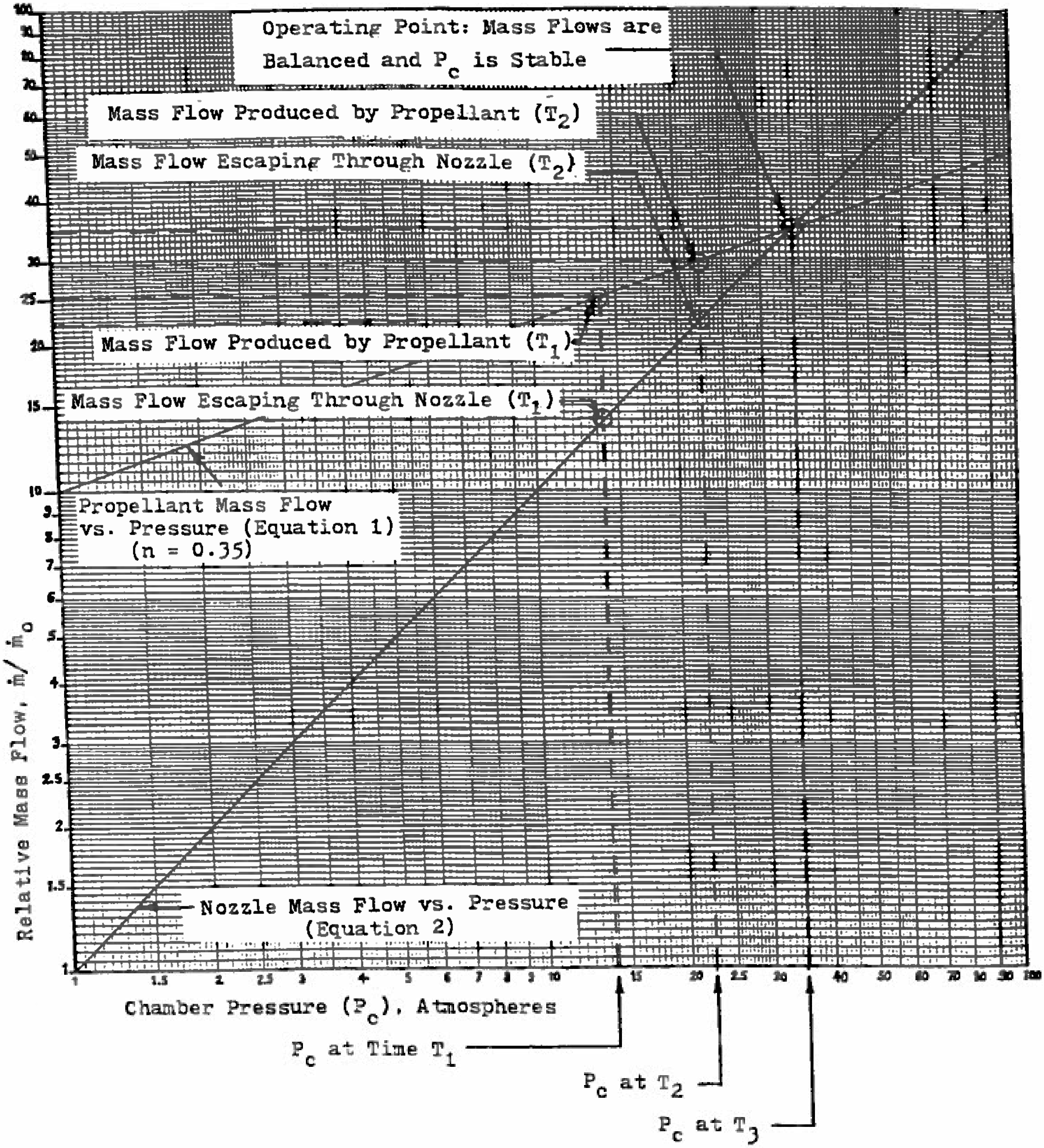


Figure 1 - Mass flow variation with chamber pressure

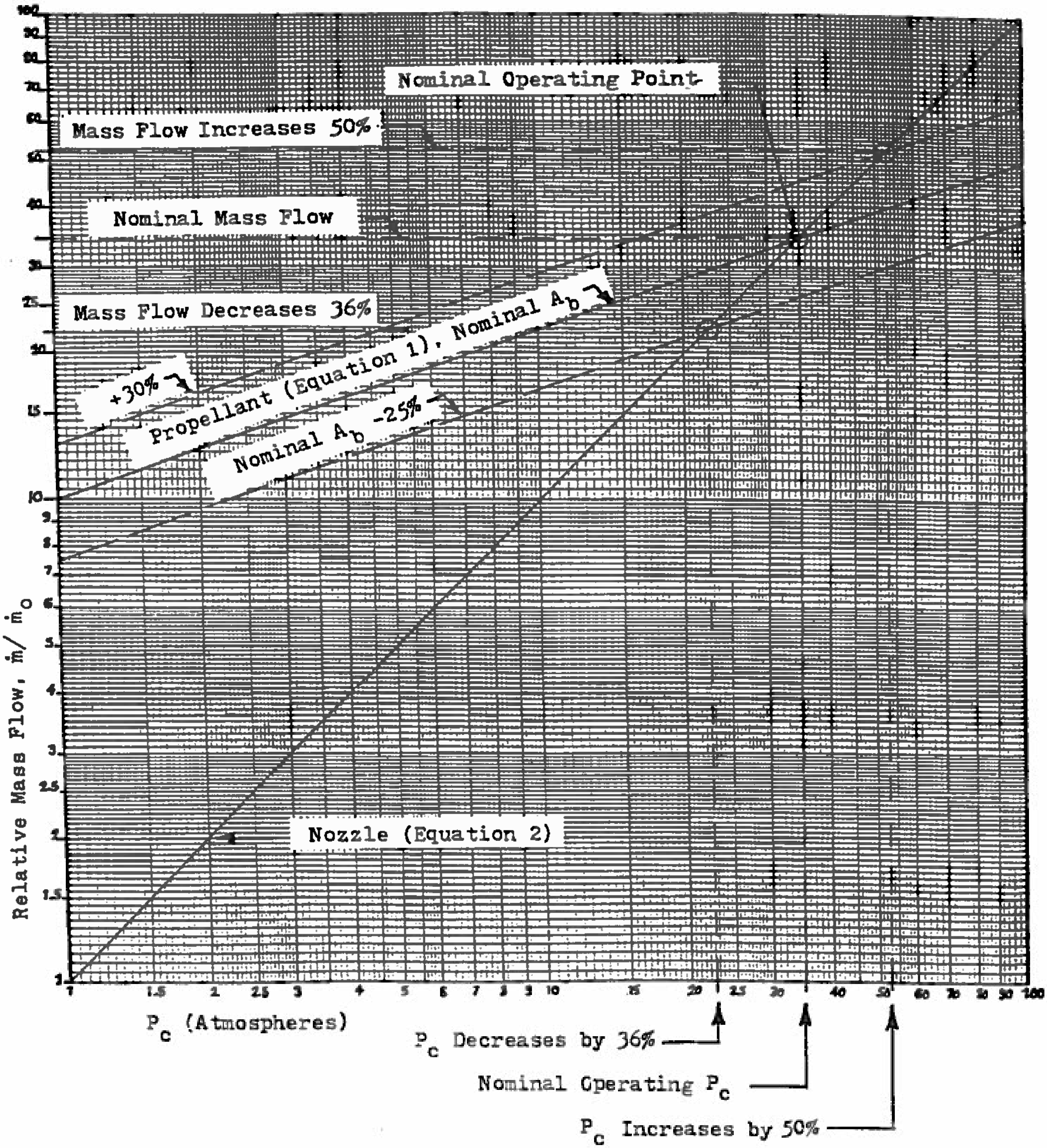


Figure 3 - Mass flow variation with chamber pressure (propellant A, n = 0.35)

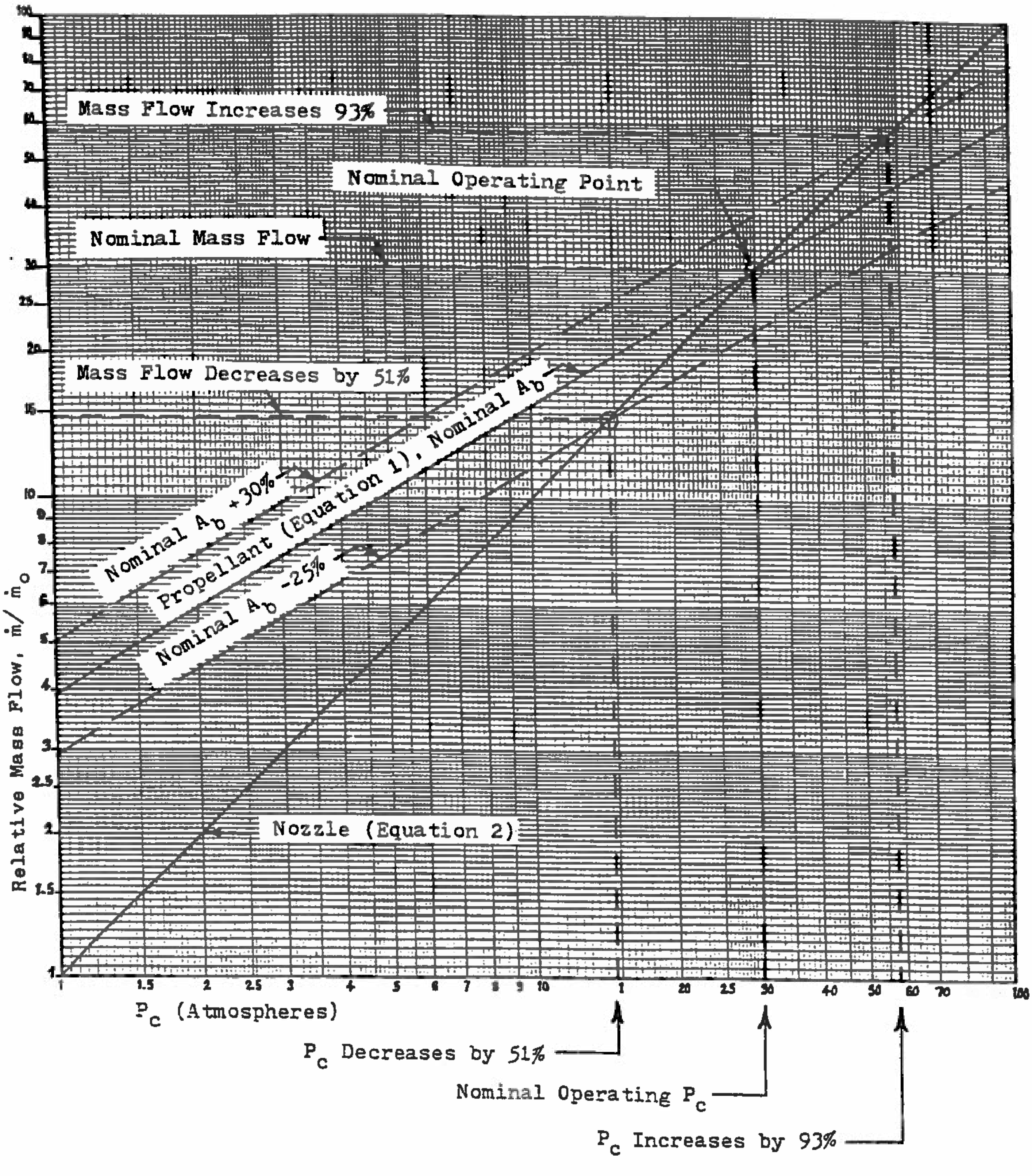


Figure 4 - Mass flow variation with chamber pressure (propellant B, n = 0.6)

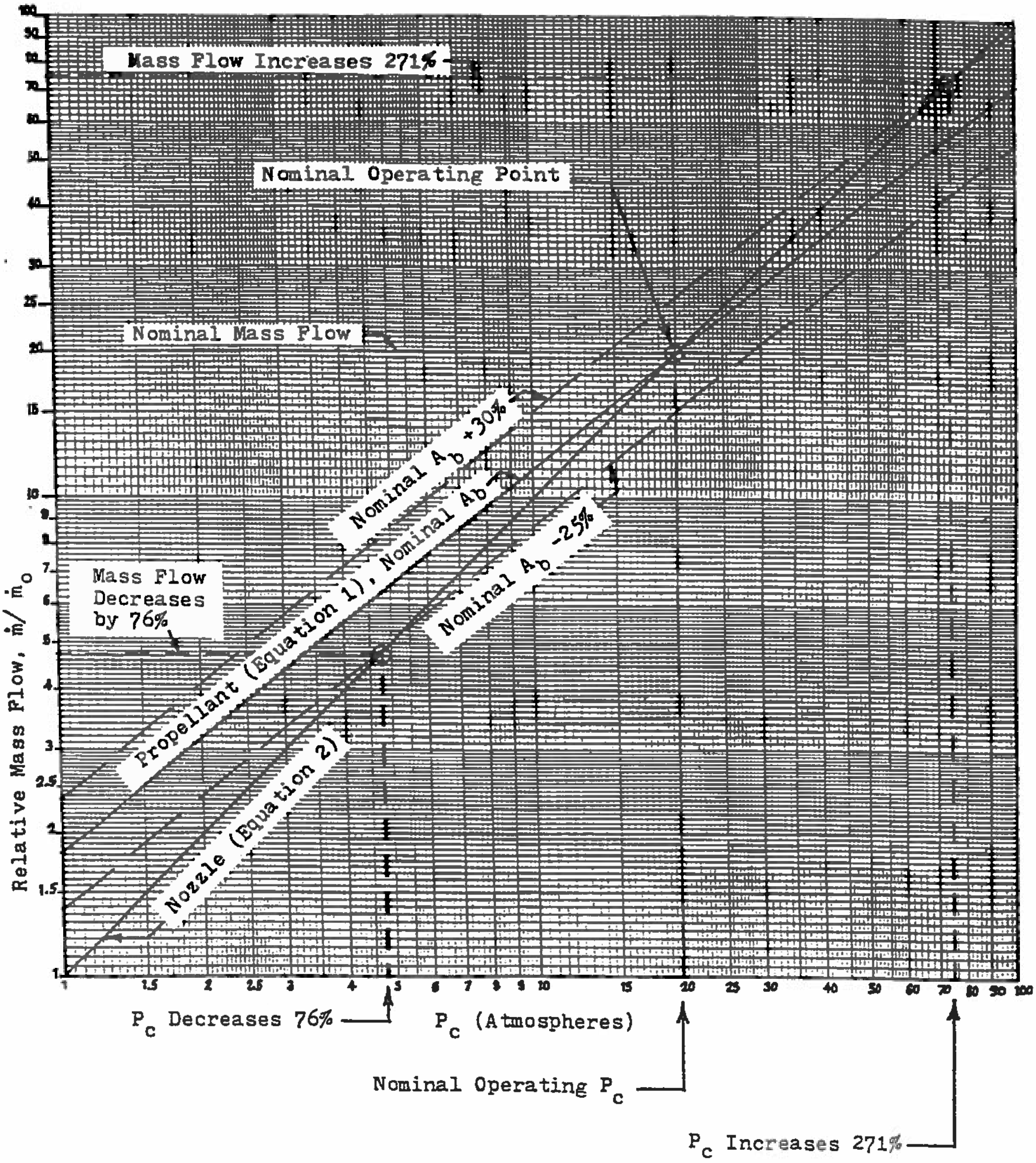


Figure 5 - Mass flow variation with chamber pressure (propellant C, n = 0.8)

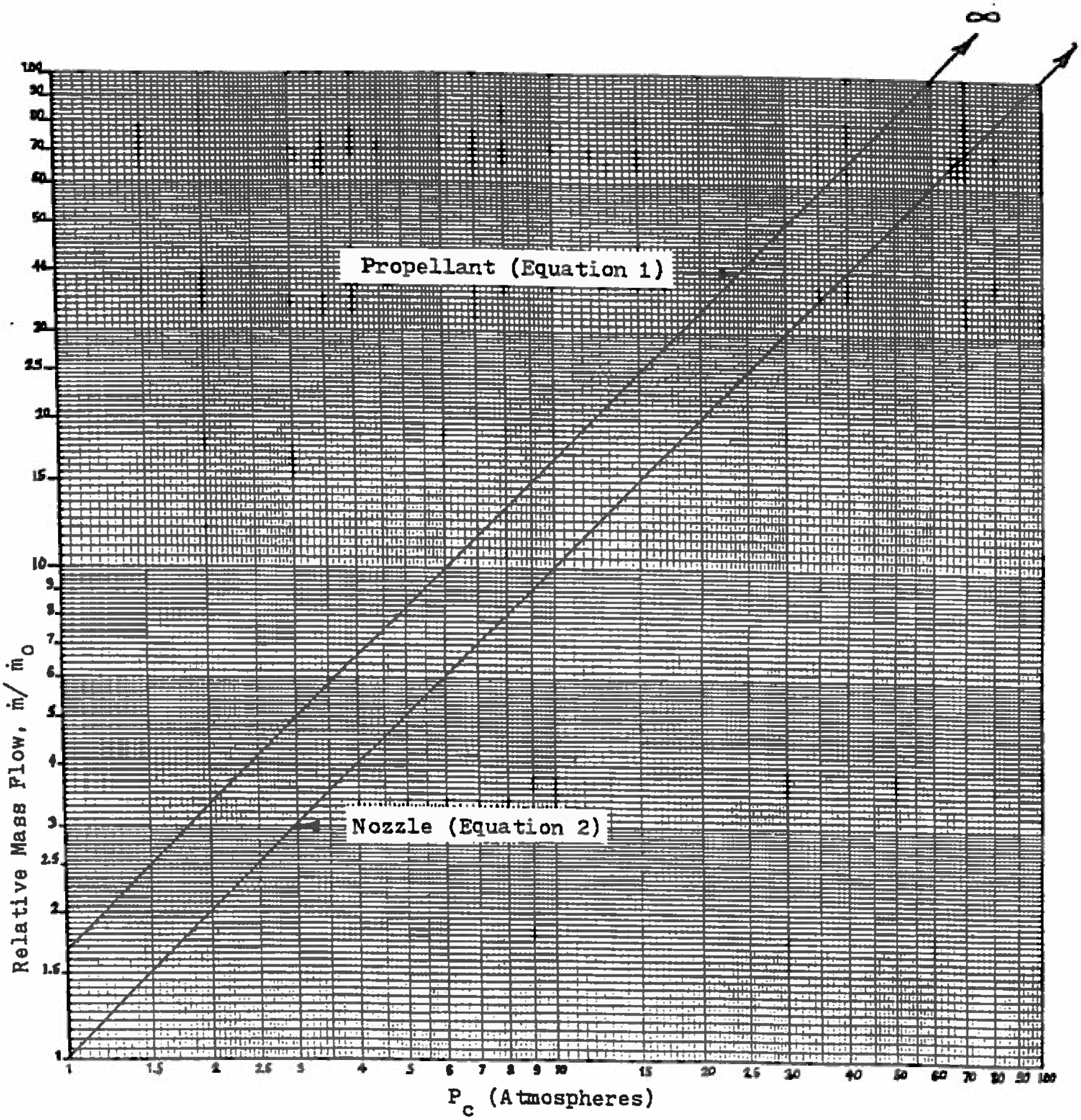


Figure 6 - Mass flow variation with chamber pressure (propellant D, n = 1.0). Pressure and mass flow increase without bound (motor explodes).

the change in pressure approaches infinity. Such a motor would be dangerous and prone to exploding. And as n gets closer to 1.0, individual motors of the same design will vary more and more in their performance, as the motor becomes increasingly sensitive to any variations in the propellant mixing or casting, initial temperature before firing, nozzle throat diameter, etc.

With an exponent of 1.0 or greater, explosion would be guaranteed.

By carefully looking at the graphs, you can see why choosing a propellant with a low value of n is desirable. As the propellant burns, any accidental pores or cracks in the grain will increase the burning area. With a low value of n, the resulting pressure increase is minimized This reduces the chances of a mechanical failure of the motor casing, nozzle, or forward bulkhead.

Thus, the burning characteristics of an ideal rocket propellant are the opposite of an ideal bomb composition or explosive.

For an AP/ HTPB type of propellant, the value of n should be somewhere around 0.3 to 0.4. Thus, it is a relatively safe, reliable, and consistent propellant in addition to having a very healthy value of Isp (specific impulse). However, it is essential for anyone designing and building his own rocket motor to first research the burning rate characteristics of the propellant he is considering, then carefully consider the implications of those characteristics.

Usually an approximate value or range of values of n can be found in published or unpublished literature. The RRS with its collection of literature, and professional and amateur rocket engineers, can be very helpful to you.

ABBREVIATIONS AND TECHNICAL TERMS

Kg = Kilograms = 2.20462 pounds (mass)
Kg/sec = Kilograms per second
m^2 = Square Meters (1 m^2 = 10.7639 $feet^2$)
m^3 = Cubic Meters (1 m^3 = 35.3147 $feet^3$)
N = Newtons (4.448 N = 1 pound force)
J = Joules (1 Watt-second = 1 Joule = 0.7376 foot-pounds)
°K = Degrees Kelvin (0°C + 273.16)
Pa = Pascal = 1 Newton per square meter
100 KPa = 100,000 Pascals = 14.5 psi = approx. 1 atmosphere
Atm. = Atmosphere = 14.696 psia
100 atmospheres = 1,469.6 Psi absolute pressure
AP = Ammonium Perchlorate (Oxidizer)
HTPB = Hydroxyl Terminated Polybutadiene (Rubbery polymer when cured)

Combustion Products = Gases, liquid droplets, and solid particles produced by combustion. Also known as flame and smoke (as a whole). The term is usually taken to mean all of the individual chemical compounds and elements that appear in the rocket exhaust.

Empirical = Calculated from experimental data, as opposed to derived from theory.

Mass Flow (m) = Flow rate of mass per unit time; = volume flow rate multiplied by density at a given point.

Resistor-Initiated Igniters

by Bob Dahlquist

Safe and very reliable igniters can be made from ordinary resistors, such as are used in electronic circuits. These igniters require substantially more voltage and current than electric matches to fire, which is a safety feature. The resistor igniter, when correctly made, is not sensitive to static electricity or electrostatic discharge.

The higher wattage used in firing the resistor igniter translates into more heat released in the initiation of ignition, which makes ignition more reliable. Even with no pyrogen dip or other pyrotechnic compound, the resistor igniter will emit a burst of flame.

When coated with an appropriate pyrotechnic compound, the resistor igniter will reliably produce a ball of fire and ignite the rocket motor. It can also be used with a powdered pyrotechnic mixture such as AlClO, but in this case, the powder may still be sensitive to electrostatic discharge although the resistor igniter itself is not.

The heart of the resistor igniter is a quarter-watt, carbon-film 5% resistor. Its value in ohms is chosen so that it will draw enough current to produce a 200x overload in the resistor. For a quarter-watt resistor, this means that when the firing button is pushed, 50 watts or more will be dissipated in the resistor. This overload converts the resistor's coating to flames within a fraction of a second.

Experiments show that the energy required to produce these flames amounted to about 14 Joules (14 Watt-seconds). The time required for flames to appear varied inversely with the average power dissipation.

$$t = (14 \text{ Watt-sec})/P_{avg} \qquad (1)$$

where t = time delay in seconds for flames to appear
P_{avg} = average power dissipation in Watts

When the resistors were immersed in black powder or coated with a pyrogen composition, the time delay for ignition was substantially less.

Carbon composition resistors were also tested, and had much longer delays. This is because the carbon, where the heat is released, is covered by a thick layer of insulating material (See Figure 1), which delays the release of heat to the exterior.

Carbon composition resistors also often broke in half without producing flame. Carbon film resistors have a ceramic substrate which holds the resistor together during and after firing. Conceivably they could be re-coated with carbon black and used again as resistors (though not as igniters).

Flameproof resistors, holding true to their name, produced no flame at all, even at extreme overloads.

Because carbon has a negative temperature coefficient (its resistance decreases with increasing temperature and vice versa - see Note 1), the resistance of the carbon film resistor igniter will decrease to about half of its initial value during firing. Average power must be used in equation 1 because the power dissipation varies during firing.

When computing power dissipation, all the other resistances in the circuit must be taken into account. This includes the internal resistance of the battery or other power source, the resistance of the wires, and the contact resistance of all the connections and relay and switch contacts; and last, but not least, the contact resistance of the igniter clips.

To achieve the most reliable ignition and shortest delay, when using carbon resistor igniters, the other resistances in the circuit should not add up to more than half of the igniter resistor's initial value. Then when the igniter's resistance decreases during firing, the other resistances in the circuit will not be greater than the igniter. (See Note 2)

In choosing what value of resistor to use, then, the criteria are:

(1) The resistor must draw enough current so that it dissipates at least 200 times its power rating initially.

(2) The value of the igniter resistor, in ohms, must be at least twice the total of all other resistances in the circuit.

(3) You must use a standard value of resistance (see Table 2) unless you are making your own resistors.

(4) For shorter ignition delay, the resistor should draw 400 times its power rating when hot (assume its hot resistance is half the initial resistance).

Table 1 gives a range of values that will meet these criteria for a 12.6 volt system.

The resistor igniter works by converting a substantial amount of electrical energy into heat in a small area. The surface area of the carbon film in a 1/4 watt resistor is roughly 5 mm^2 or 0.05 cm^2. The rate of heat release at 100 watts dissipation is 23.9 (gram) calories per second. The heat flux in the 1/4 watt resistor igniter at 100 watts is thus approximately

$$\text{Heat flux} = (23.9\ \text{cal/sec})/(0.05\ \text{cm}^2)$$
$$= 478\ \text{cal/cm}^2\text{- sec.}$$

In theory, this is almost 16 times the heat flux required to ignite HTPB propellant directly (see note 3), if the resistor were cast into the propellant or inserted into a close-fitting hole or slot; and it has been verified by experiment that the resistor will indeed ignite the propellant directly. But, to have a short ignition delay within the motor as a whole, a faster burning pyrotechnic compound is usually needed as a booster.

The flame from the resistor lasts longer than the flame produced by an electric match, giving time for more heat energy to transfer to the pyrotechnic booster compound. This makes the ignition sequence more foolproof and allows less sensitive

Table 1: Resistance values for 12.6 volt system

To Function Reliably			For Shorter Ignition Delay		
R_i	Max. R_c	Max. R_t	R_i	Max R_c	Max. R_t
1.0	0.50	1.50	1.0	0.391	1.391
1.1	0.55	1.65	1.1	0.384	1.484
1.2	0.60	1.80	1.2	0.376	1.576
1.3	0.65	1.95	1.3	0.366	1.666
1.5	0.68	2.18	1.5	0.340	1.840
1.8	0.59	2.39	1.8	0.295	2.095
2.0	0.52	2.52	2.0	0.260	2.260
2.2	0.44	2.64	2.2	0.220	2.420

All values are in ohms.
R_i higher than 2.2 is not recommended for 12.6 volts.
Left columns are for $P_i = 200 * P_r = 50$ Watts minimum.
Right columns are for $P_h = 400 * P_r = 100$ Watts. (1/4 Ohm Resistor)

R_i = Resistor initial value
P_i = Initial dissipation
P_r = Resistor power rating
R_c = Circuit resistance
P_h = Hot dissipation
R_t = Total resistance

Table 2: Standard 5% resistor values useful for ignition (ohms)

1.0	2.7	6.8	18	47
1.1	3.0	7.5	20	51
1.2	3.3	8.2	22	56
1.3	3.6	9.1	24	62
1.5	3.9	10	27	68
1.6	4.3	11	30	75
1.8	4.7	12	33	82
2.0	5.1	13	36	91
2.2	5.6	15	39	100
2.4	6.2	16	43	110

booster compounds to be used.

The disadvantage is that the energy source required for the resistor igniter is much larger and heavier. Thus, the resistor igniter is not practical for second-stage ignition, parachute deployment, etc, except in rather large rockets. For such purposes, more sensitive igniters are still needed.

For short ignition delay using resistor igniters, the power source must have low internal resistance. Note that R_C in Table 1 includes the internal resistance of the power source in addition to the resistance of igniter leads, system wiring, and contacts.

When R_C is low, the resistor undergoes a thermal runaway effect during firing. As the resistor initially conducts electricity and begins to heat up, its resistance decreases due to carbon's negative temperature coefficient. This causes more current to flow, which generates more heat, which causes the resistance to decrease further, which causes more current to flow, and so on.

The thermal runaway effect shortens the ignition delay of the carbon resistor igniter. If R_C is not low, then R_C tends to limit the current increase, thus preventing the thermal runaway. To avoid this, R_C should be no greater than the values shown on the right side of Table 1. With lower values of R_C the ignition delay will be shorter.

The maximum allowable R_C in keeping with this consideration can be calculated for any given voltage, as follows:

For short ignition delay the resistor should dissipate at least 400 * P_r when hot. We want the hot

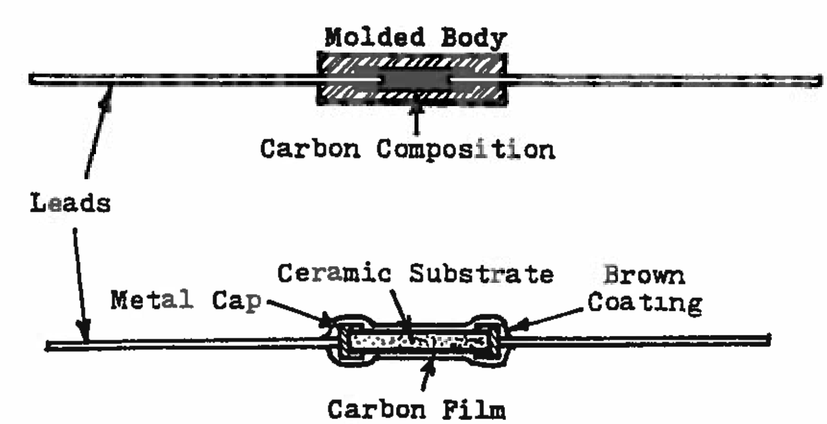


Figure 1 - Types of carbon resistors (cross sections)

resistance R_h to be 1 to 2 times R_c. Therefore, begin by assuming $R_h = R_c$ and $R_t = R_h + R_c = 2*R_h$.

Then, the total circuit power must be $2*(400 * P_r)$, or $800 * P_r$ based on the hot resistance. Half of the power will be dissipated in the circuitry and half in the resistor.

The maximum R_t is given by

$$R_t = E^2 / P_t \qquad (2)$$

If P_r is 1/4 watt, then

$$R_t = E^2 /200, \qquad (3)$$

where E is the open-circuit voltage and R_t is total resistance and the maximum R_c is:

$$R_c = E^2/400 = R_h \qquad (4)$$

For less ignition delay, limit R_c to half the value given by Equation 4, or use:

$$R_c = E^2/800 = (1/2) * R_h \qquad (5)$$

Now, the value of R_h from Equation 4 must be doubled to find R_i and then R_i (the initial resistor value) must be rounded off to a standard value (see Table 2).

After rounding off, a new maximum value for R_c should be calculated based on the new values of R_i and R_h, as follows.

Calculate a value of current that will cause $400 * P_r$ to be dissipated in the new value of R_h (which is 1/2 the new value of R_i).

$$\begin{aligned} I &= (P/R_h)^{0.5} \\ &= (400 * P_r /R_h)^{0.5} \qquad (6) \end{aligned}$$

where I = Current (amps) required (minimum)
P = Power dissipated in the resistor, watts
P_r= Power rating of the resistor (1/4 watt)
R_h = Hot resistance of the resistor, ohms

Now, use Ohm's Law to calculate total resistance R_t such that the desired current will flow:

$$R_t = E/I \qquad (7)$$

The new value of R_c is then found by subtraction:

$$R_c = R_t - R_h \qquad (8)$$

This is a maximum value for R_c; using a lower value will result in a shorter ignition delay.

Example 1: The power source for this example is a 9.6 volt Nicad battery pack.

Assume: $R_h = R_c = E^2/400 \qquad (4)$

$$R_h = R_c = 92.16/400 = 0.23 \text{ ohm}$$

$$R_i = 2 * R_h = 0.46 \text{ ohm}$$

Since the lowest standard value is 1.0 ohm, R_i must be rounded off to 1.0 ohm. The new R_h is half of that or 0.5 ohm.

The minimum current required for $400 * P_r$ is:

$$\begin{aligned} I &= (400 * P_r/R_h)^{0.5} \\ &= (400 * (1/4) / 0.5)^{0.5} \\ &= (200)^{0.5} = 14.14 \text{ amps} \qquad (6) \end{aligned}$$

The maximum total resistance R_t for this current to flow is:

$$\begin{aligned} R_t &= E/I = 9.6 \text{ volts}/14.14 \text{ amps} \\ &= 0.6788 \text{ ohm} \qquad (7) \end{aligned}$$

The new value of R_c is:

$$\begin{aligned} R_c &= R_t - R_h = 0.6788 - 0.500 \\ &= 0.1788 \text{ ohm (max.)} \qquad (8) \end{aligned}$$

Note that R_c includes the internal resistance of the battery pack. Note that 0.1788 ohm is a maximum; a lower value will result in a shorter ignition delay.

What is the absolute minimum voltage to be used with a resistor igniter? Assume R_c is zero and R_i is the minimum standard value, 1.0 ohm. Then,

$P_i = 200 * P_r$, based on R_i, and $P_r =$ 0.25 watt. Then, $P_i =$ 50 and

$$E = (P_i * R_i)^{0.5} = (50)^{0.5} = 7.07 \text{ volts}$$

or, $P_h = 400 * P_r$ and $R_h = 0.5$ ohm. Then, $P_h =$ 100 watts and

$$E = (P_h * R_h)^{0.5} = (50)^{0.5} = 7.07 \text{ volts.}$$

In practice, this voltage is too low because R_c will never be zero except in a laboratory situation with a regulated power supply and a four-wire connection to the igniter resistor. But it is useful to know what the lower limit is. To find the practical lower limit, multiply the absolute lower limit by $(2)^{0.5}$ to allow for circuit resistance, and round off to the nearest available battery voltage.

$7.07 \times (2)^{0.5} = 10$ volts (9.6 volts is close)

The following example involves the highest practical voltage for use of a resistor igniter. The power source for this example is a portable generator producing 120 volts AC at 1,000 feet from the rocket, on a very large dry lake bed.

Example 2: Assume E is 120 volts and Pr is 0.25 watt. What value of resistor should be used, and what gauge wire is required for a cable length of 1000 feet?

Assume: $R_h = R_c = E^2/400$ (4)

$$R_h = R_c = 14400/400 = 36 \text{ ohms}$$

$$R_i = 2\,R_h = 72 \text{ ohms}$$

Round this off to 75 ohms, the nearest standard value (Table 2).

The new R_h is 75/2, or, 37.5 ohms.The minimum current required for $400 * P_r$ watts is

$$I = (400 * P_r/R_h)^{0.5}$$
$$= (400 * 0.25/37.5)^{0.5} = 1.633 \text{ ohms} \qquad (6)$$

The maximum total resistance R_t for this current to flow is

$$R_t = E/I \qquad (7)$$
$$= 120 \text{ volts}/1.633 \text{ amps} = 73.48 \text{ ohms}$$

The new value of Rc is then

$$R_c = R_t - R_h \qquad (8)$$
$$= 73.48 - 37.5 = 35.98 \text{ ohms}$$

Round off to 36 ohms in this case. Normally, you would round down, not up.

Check the copper wire table to see what size wire has low enough resistance (Table 3). Remember that there are 2000 feet of wire in a 1000 foot pair. Also don't forget to allow for the internal resistance or droop of the power source.

Because the total length of wire is 2 x 1000 feet, its resistance per 1000 ft. must be no more than 36/2, or 18 ohms/1000 feet. According to the Table 3, #22 has 16.46 ohms/1000 feet and will carry 8 amps. A #22 pair would have about 33 ohms of resistance, and will work if the internal resistance power source is 3 ohms or less. A #20 pair would have 20.7 ohms of resistance and would reduce the ignition delay

Note that when using 120 volts AC, the insulation of both the cable and the igniter leads must be able to withstand the peak voltage. With a square wave source such as a cheap inverter, the peak voltage is 120 volts. With a sine wave source such as a generator, it is $(2)^{0.5} * 120$, or 170 volts.

When using voltage above about 40, it is possible to have an arc in the igniter or between the igniter leads after ignition. It is also possible for the igniter leads to weld themselves together. An arc has relatively low resistance, and of course a weld has essentially no resistance. Thus the current would be limited by the R_c alone.

Table 3 - Resistance of copper wires

Gauge (AWG #)	Milliohms Per Foot or Ohms Per 1000 Feet of Single Wire	Current Carrying Capacity, Amps Continuous
10	1.018	55
12	1.610	41
14	2.575	32
16	4.094	22
18	6.510	16
20	10.35	11
22	16.46	8
24	26.17	5.5
26	41.62	4
28	66.17	2.75
30	105.2	2

For a pair of wires, multiply the length of the pair by 2.
At 1 amp, each milliohm causes one millivolt of voltage drop.
At 10 amps, 100 milliohms would produce a voltage drop of 1 volt.

In this case our R_C is somewhat greater than 33 ohms. Then, the short circuit current will be limited to:

$$I = E/R = 120 \text{ volts}/33 \text{ ohms} = 3.64 \text{ amps}$$
(with #22 cable)

When using voltages over 40, make sure your R_C is not too low; add a ballast resistor or current limiting fuse, if necessary.

If other things such as fuel and LOX valves and computers are running off the same power source, you do not want shorted igniter leads to trip the main circuit breaker. If you want to see some very nervous people, just attend a launch where this happens and leaves a fully loaded and pressurized liquid fuel rocket sitting on the pad with no way to vent the LOX tank until someone goes out and gets the power source on line again.

The 120 volt igniter circuit should have its own properly sized magnetic circuit breaker or current limiting, fast acting fuse, or its own separate power source.
In addition, the white wire should not be grounded except through a 1-megohm resistor, unless a ground fault circuit interrupter (GFCI) is used, for personnel safety.

Internal Resistance

The internal resistance or effective series resistance of any power source can be calculated after first measuring the voltage droop under load:

$$\text{Res} = \Delta E / I$$

where ΔE = voltage droop
I = current at which ΔE is measured
Res = effective series resistance, or internal resistance of the power source being tested.

Assembly

Once you have chosen the right value of resistor, you can assemble your resistor igniters as shown in Figure 2.

You must choose the appropriate igniter lead wire

.6 grams of Al KClO4 of Aluminum

gauge for the length of leads that you need. The igniter leads are part of R_c, so don't forget to include their resistance in your calculations. The drawing shows #24 wire 18 to 36 inches long, but you can use whatever you need. (Refer to Table 3.)

Notes

1. The temperature coefficient of resistivity for graphite varies with temperature, but on average it is about - 2.7 x 10^{-4} ohms/ohm-°C, that is, the resistivity decreases as the temperature increases.

To find the hot resistance, R_h, you can use the following formula.

$R_h = R_i + R_i * (T_h - T_i) * (-2.7 \times 10^{-4})$ ohms

where R_i = the cold resistance in ohms
T_i = the initial temperature in °C
T_h = the hot temperature in °C

2. For a given value of EMF and R_c, maximum power to the load resistance occurs when $R_L = R_c$. When R_L is less than R_c, most of the power is dissipated in R_c (defining R_L as the load resistance or igniter resistor, and R_c as all the other resistance in the circuit combined).

3. The approximate threshold heat flux is given as 30 cal/cm2-sec in Solid Rocket Ignition by Lawrence G. Teebken, RRS News Vol 53, #3 - Sept., 1996 (at page 29).

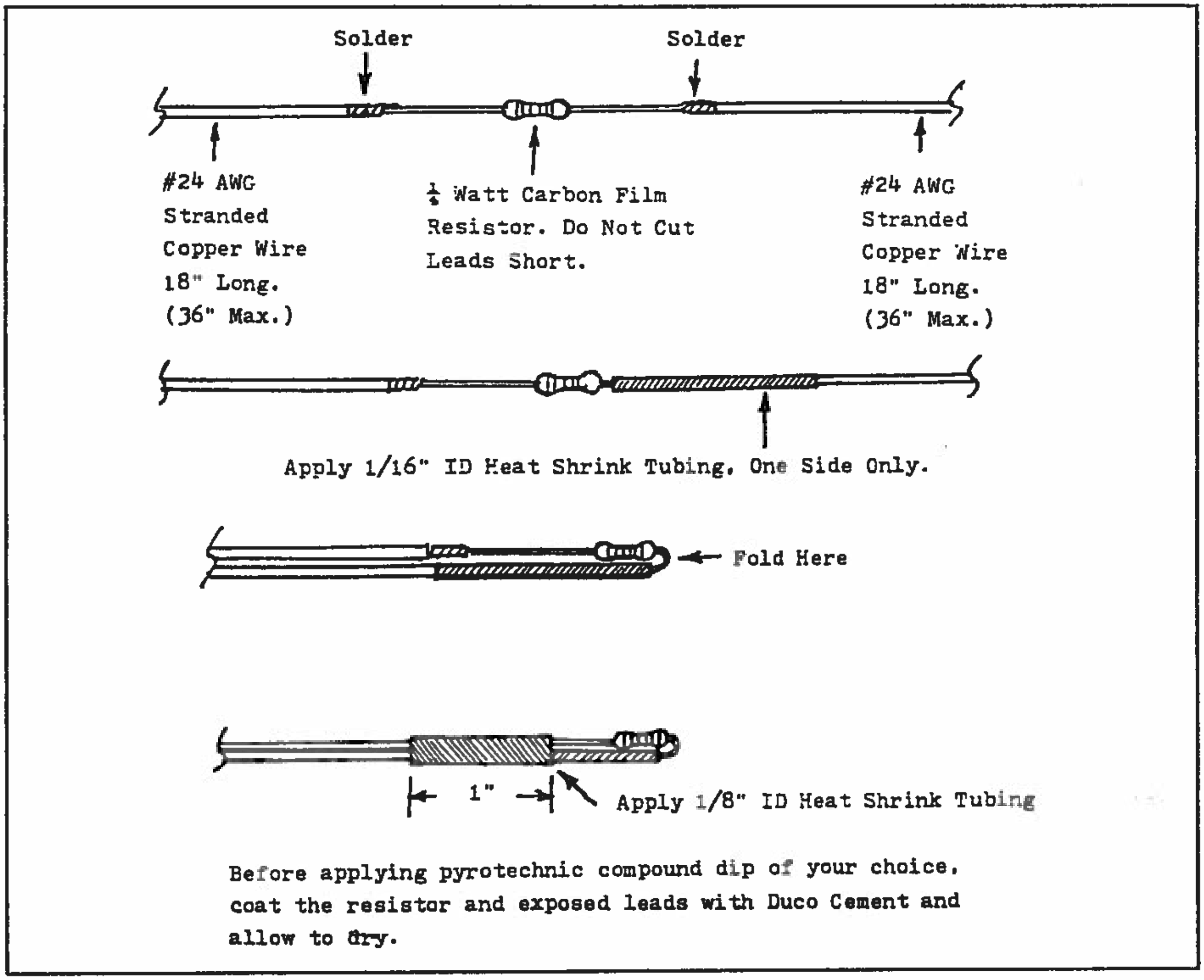


Figure 2 - Assembly of a resistor igniter