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Model Rocket Motors, Theory and Design[1]

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ABSTRACT

A semi-empirical theory is presented for the design of model rocket motors that use Black Powder for fuel. By choosing the values of a few adjustable parameters, a hobbyist can construct motors that perform satisfactorily without extensive or dangerous trial and error. Formulas are given for calculating the nozzle diameter, the combustion chamber height, and such performance descriptors as specific impulse and average thrust for any size of model rocket motor.

Key Words: model rocketry, Black Powder, rocket propulsion theory

Introduction

For a beginner, building model rocket motors without any technical knowledge is a hazardous pursuit. Because of this, the competition rules of the international model rocket association, Federation Aeronautique Internationale (FAI), prohibit the use of any motors that do not meet certified safety standards. Usually, the only motors that qualify are commercial ones. And even they can be dangerous if they are modified by the modeller or they are not used according to the manufacturer's instructions. Experimental model rocket motors of any kind introduce considerably higher risks. And those that are constructed only "by the seat of one's pants" are disasters waiting to happen. In order to use factory-made or hand-made motors with maximum safety, one must thoroughly understand and conscientiously apply the principles of rocket propulsion.

Theory

We will develop a semi-empirical theory[2] for the most common form of model rocket motor, namely one in which a solid fuel is compressed or molded into a cylindrical casing. Although the theory is quite general and applies to any cylindrically-shaped solid fuel, we will cite the adjustable parameters in ranges that pertain to Black Powder only.

The theory centers around two unitless parameters, f_{max} and f_{min}, which effectively tailor the internal geometry of the rocket motor to the power of the particular fuel. Or, alternatively, they specify what strength of fuel should be used for a fixed internal shape. The first parameter f_{max} is the ratio of the maximum burning area to the nozzle area. It can be as low as 44 for high-power, military-quality Black Powder, or it can be as high as 100 or more for haphazardly-mixed, hand-made meal. The other parameter, f_{min} compares the area of minimum burning to the area of the nozzle. It ranges from 18 to 50.

The two numbers can be optimized with extensive testing, but in practice, they usually lead to acceptable rocket performance if they each can be determined within ± 10 of their ideal values. And this can be accomplished with only a few test motors. One either chooses values for several f_{max} and f_{min} pairs based on an estimate of the fuel's power, then

makes prototype motors corresponding to each pair to see which works best. Or, for an existing set of rocket-making tools, one adjusts the fuel mixture to get f_{max} and f_{min} values which match the tools' geometry. Either way, an amateur rocket builder can design reliable motors with a minimum of trial and error.

Combustion Intervals

The performance of a motor over time can be divided into two intervals: (1) the *lift-off* interval, where the motor develops the necessary surge of thrust to get itself and its payload off the ground, and (2) the *end-burning* interval, where the motor maintains its upward movement with a steady thrust. The effects of these two intervals can be visualized in Figure 1 where the thrust of this type of model rocket motor is plotted against time.

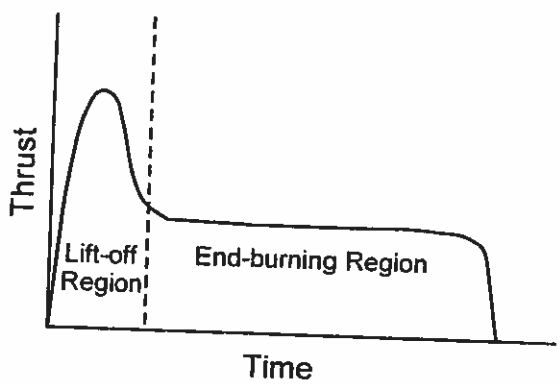

Figure 1. Combustion intervals for a solid-fuel rocket motor with a hollow combustion cavity.

The Lift-Off Region

The amount of thrust that a burning fuel can supply is directly proportional to the surface area of combustion. If a rocket motor is fueled with nothing more than a solid cylindrical mass of Black Powder, it only has the circular cross-sectional area of the cylinder as its burning surface. And an area that small is often insufficient to provide the requisite lift-off thrust. Thus, a hollow cavity of some shape is usually made in the fuel cartridge in order to increase the initial surface area of combustion.

Ignition causes burning over this larger cavity surface. As the combustion progresses, fuel is consumed. Consequently, the size of the cavity becomes larger still, and the burning surface grows. Greater volumes of gas are forced through the nozzle, and the rocket is propelled upward by ever-increasing forces. At some point, however, the burning surface becomes as large as it possibly can. This is the point that defines f_{max}, the ratio of the maximum combustion area A_{max} and the nozzle area A_{noz}:

$$f_{max} = \frac{A_{max}}{A_{noz}} \tag{1}$$

Cylindrical Cavity

Consider the case where the rocket motor has a cylindrical combustion cavity. Let the motor have an inside diameter d, a cavity height h_{cyl}, and a nozzle diameter n (Figure 2).

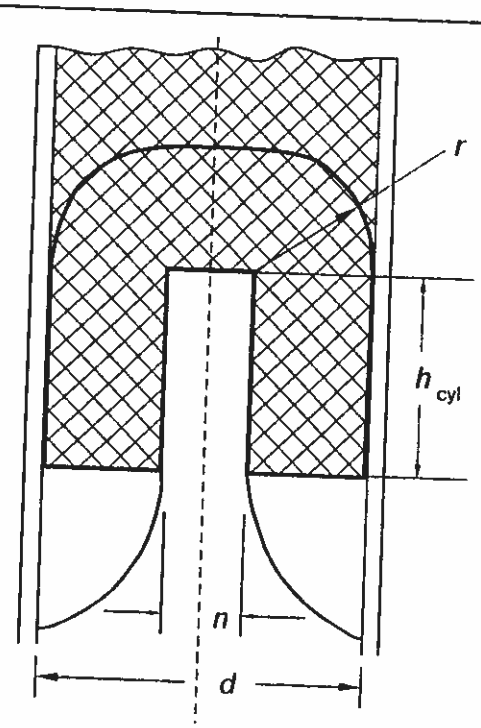

Figure 2. Cross section of a rocket motor with a cylindrical combustion cavity.

On ignition, the fuel begins to burn away from the cavity's initial surface. We assume that it is consumed in parallel, equidistant layers. When it has burned outward (and simultaneously upward) a distance, $r = \frac{1}{2}(d-n)$, it cannot go any farther laterally. At that moment it reaches its maximum combustion area. This surface is the sum of the area above the nozzle, the area of the rounded edges[3], and the area of cylindrical sides. Thus,

$$A_{max} = \frac{\pi n^2}{4} + \left(2\pi r^2 + \frac{\pi^2 n r}{2}\right) + \pi d h_{cyl}$$

$$A_{noz} = \frac{\pi n^2}{4} \qquad (2)$$

and

$$f_{max} = 1 + \frac{8 r^2 + 2\pi n r + 4 d h_{cyl}}{n^2} \qquad (3)$$

From this, we find the height of the combustion chamber:

$$h_{cyl} = \frac{d\left(f_{max} - 2\alpha^2 - 3 + \pi\right)}{4\alpha^2} + \left(1 - \frac{\pi}{4}\right) n \qquad (4)$$

where $\alpha = d/n$ is the important ratio of casing diameter to nozzle diameter.

As we shall see in equation 21 below, $\alpha^2 = f_{min}$ is the other principal parameter. Thus,

$$h_{cyl} = \frac{d\left(f_{max} - 2 f_{min} - 3 + \pi\right)}{4 f_{min}} + \left(1 - \frac{\pi}{4}\right) n \qquad (5)$$

Whether or not the fuel burns in a precise parallel manner, as we have assumed, does not matter. The reason is the semi-empirical way in which f_{max} and f_{min} are treated. Their values are *chosen*, based on some guidelines and on some testing. Therefore, when such experimentally-adjusted parameters are used in the formula, the resulting chamber height becomes as accurate as the user wants it to be.

Conical Cavity

If the spindle tool has a slight taper (for easier removal after ramming), then the combustion cavity becomes the frustrum of a cone as in Figure 3. Here, the height of the cavity is h_{con}, and the narrower diameter at the top of the cavity is δ. The maximum combustion area A_{max}, in this case, is

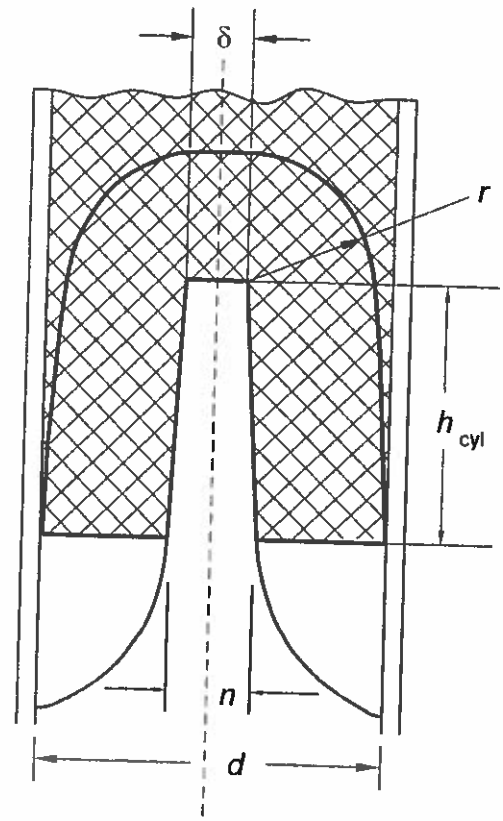

Figure 3. Cross section of a rocket motor with a conical combustion cavity.

$$A_{max} = \frac{\pi\delta^2}{4} + \left(2\pi r^2 + \frac{\pi^2 \delta r}{2}\right) + \frac{\pi\left(d^2 - \kappa^2\right)}{4}\sqrt{1 + \left(\frac{2 h_{con}}{d - \kappa}\right)^2} \qquad (6)$$

where

$$\kappa = 2 r + \delta \qquad (7)$$

The height of the conical combustion chamber is

$$h_{con} = \frac{n-\delta}{2}\sqrt{\left[\frac{n^2 f_{max} - \delta^2 - 2(d-n)\left(d-n+1/2\pi\delta\right)}{2d(n-\delta)-(n-\delta)^2}\right]^2 - 1} \qquad (8)$$

But we have specified that the conical taper is gradual. Therefore, the quantity $n-\delta$, is small, and the formula can be simplified considerably by ignoring the $(n-\delta)^2$ term and the 1. The resulting approximate chamber height is

$$h_{con} \approx \frac{d f_{max}}{4 f_{min}} - \frac{\delta^2 - 2(d-n)(d-n+1/2\pi\delta)}{4\,d} \tag{9}$$

This approximation gives values that differ from equation 8 by, at most, only a few tenths of a millimeter. And since f_{max} and f_{min} will be adjusted experimentally anyway, such differences are insignificant.

As in the cylindrical case, these semi-empirical formulas remain useful even though there may be flaws in the motor's geometry or unevenness in the fuel's burning.

The End-Burning Region

When the combustion of fuel has reached far enough up the cartridge so that the burning surface is essentially the flat cross-sectional area of the rocket tube, the thrust remains constant until the fuel is exhausted. Under these conditions, many important properties of the motor can be calculated. The necessary formulas are derived from the thermodynamics of the fuel and of its combustion products, and they contain other adjustable parameters. A fundamental explanation of thermodynamics is, of course, beyond the scope of this paper. But we will use only two abstract quantities that would require such explanations.

The first of these is k, the "heat-capacity index" for the combustion products. For solid fuel mixtures, this index has been found[4] to be of the form

$$k = 1.30\left(\frac{T_c}{273}\right)^{-0.032} \tag{10}$$

where T_c is the absolute temperature of the combustion chamber. For commercial Black Powder, T_c can be taken as 2300 K. T_c ranges from 1150–2300 K for other mixtures. But k does not vary much (1.21 to 1.24).

The other is the "gas parameter" for the combustion products R (in J/kg·K). It can vary greatly for fuels with potassium nitrate as the oxidizer. R is related to the power (or energy content) of the fuel E:

$$R = \frac{E}{k\,T_c} \tag{11}$$

And since E has values[5] in the range of 230–280 kJ/kg for Black Powder, the resulting R's can be anywhere from 100 to 160.

With these two parameters calculated, we can now derive some of the quantities that describe the rocket motor's performance.

The whole point of rocketry is to create a gas pressure P_c in the combustion chamber that is large enough to move the motor but not large enough to cause an explosion. P_c (in Pa) can be found with the formula

$$P_c = \frac{m_f\sqrt{RT_c}}{t \cdot b \cdot A_{noz}} \tag{12}$$

where the parameter b is a function[6] of k:

$$b = \sqrt{k\left(\frac{2}{k+1}\right)^{\frac{k+1}{k-1}}} \tag{13}$$

Since k is nearly constant, so is b; it only varies between 0.651 and 0.656.

In equation 12, m_f is the mass of fuel (in kg) to be consumed in the motor, and t is the time (in s) that the motor will operate. One of these two numbers is chosen by the motor designer. The other is then specified within the following calculations:

The combustion velocity U, in m/s, (which depends on P_c) is

$$U = U_o\left(P_c \times 10^{-5}\right)^{\nu} \tag{14}$$

where U_o, the combustion velocity at 0.1 MPa (1 atm), varies from 8.8–12.1 mm/s (0.0088–0.0121 m/s), and ν, the pressure exponent, ranges from 0.5 to 0.24. U_o and ν can be taken as 12.1 and 0.24, respectively, for commercial Black Powder. The end-burning area A_{end} is

then found as a function of U and of the density of the fuel ρ:

$$A_{end} = \frac{\pi d^2}{4} = \frac{m_f}{U \cdot t \cdot \rho} \tag{15}$$

The fuel density depends on how the rocket motor is constructed (see Table 1).

Table 1. The Density of Black Powder Fuel Resulting from Various Construction Techniques.

Fuel Compaction Method	ρ (g/cm^3)
Ramming	1.2–1.3
Screw press	1.3–1.5
Hydraulic press	1.5–1.8

The specific impulse I_{sp}, in s is the standard measure of a rocket's power.[7] It is calculated with the formula

$$I_{sp} = \frac{1}{g}\sqrt{\frac{2k}{(k-1)} R \left(T_c - T_o\right)} \tag{16}$$

where g is the gravitational constant, 9.8 m/s^2 and T_o is the temperature of the combustion gases after they have escaped the nozzle

$$T_o = T_c \left(\frac{P_o}{P_c}\right)^{\frac{k-1}{k}} \tag{17}$$

and where P_o is taken as atmospheric pressure, or 0.1 MPa (1 atm).

The average thrust or propulsion force F_{ave}, in N, is

$$F_{ave} = \frac{m_f g I_{sp}}{t} \tag{18}$$

The total impulse I_{tot} is of great importance in classifying the motor for FAI competition:

$$I_{tot} = t F_{ave} = m_f g I_{sp} \tag{19}$$

Finally, the nozzle diameter can be calculated in terms of f_{min}, the ratio of the minimum combustion area $A_{min} = A_{end} = \frac{1}{4}\pi d^2$ to the nozzle area $A_{noz} = \frac{1}{4}\pi n^2$. Thus,

$$n = d\sqrt{\frac{1}{f_{min}}} \tag{20}$$

This verifies the assertion that we made earlier in the formulas for the heights of the variously-shaped combustion chambers:

$$\alpha^2 = \frac{d^2}{n^2} = f_{min} \tag{21}$$

We have now presented everything necessary for anyone to design a Black-Powder-fueled, model-rocket motor. We summarize, in Table 2, all the adjustable parameters used in this theory together with workable upper and lower bounds for their Black Powder values. These values are given in SI units, but any consistent set of units may be used.

Sample Calculations

Suppose we wish to design a rocket motor with an inside diameter of 0.012 meters (half an inch), and we want it to operate for 2 seconds. We anticipate, of course, that different fuel mixtures will produce different results. To get a feel for the possibilities, let us calculate the properties of two example motors using Black Powder of vastly disparate power. In the first example, we will use some very good commercial Black Powder, perhaps military surplus, for which all the values in the last column of Table 2 apply. We will compact it with a hydraulic press so that it will attain a density of 1720 kg/m^3. In the second example, we will use hand-made meal that has the parameters given in the next-to-last column of Table 2 (except f_{min} will be taken as 36). We will ram it by hand to a density of only 1270 kg/m^3.

Table 2. The Semi-Empirical Parameters Appearing in This Theory and Their Variations Depending on the Quality of Black Powder Used as Fuel.

Parameter	Symbol	Poor-Quality, Hand-Made	High-Grade, Commercial
Maximum area ratio	f_{max}	100	44
Minimum area ratio	f_{min}	50	18
Combustion temperature (K)	T_c	1150	2300
Energy content of fuel (J/kg)	E	230,000	280,000
Combustion rate at 1 atm (m/s)	U_o	0.0088	0.0121
Pressure exponent	ν	0.5	0.24
Density of the fuel (kg/m^3)	ρ	(*see Table 1*)	

Table 3. Calculated Results for a 12 mm Motor that Operates for 2 Seconds on (1) Commercial Black Powder Compressed to a Density of 1.72 g/cm^3, or (2) Hand-Made Powder Rammed to a Density of 1.27 g/cm^3.

Calculated Quantity	Symbol	Example 1	Example 2
Heat capacity index	k	1.21	1.24
Gas Parameter (J/kg·K)	R	100	161
Nozzle diameter (mm)	n	2.83	2.00
Height of cylindrical cavity (mm)	h_{cyl}	1.96	2.77
Height of conical cavity from eqn. 8 (mm)	h_{con}	2.19	2.94
Height of conical cavity from eqn. 9 (mm)	h_{con}	2.17	2.92
Ratio of casing diameter to nozzle diameter	α	4.24	6.00
Mass of fuel (g)	m_f	6.1	6.1
Pressure in combustion chamber (MPa)	P_c	0.358	0.637
Combustion velocity (mm/s)	U	16.4	22.2
Specific impulse (s)	I_{sp}	74.0	77.4
Total impulse (N s)	I_{tot}	4.43	4.63
Average thrust (N)	F_{ave}	2.21	2.31

Table 3 gives the results of the calculations. The heights of both cylindrical and conical cavities are determined in each example. (For the conical cases, δ was taken as 0.9 n). Note that in both examples, the heights of the conical cavities are greater than the cylindrical ones. That is to be expected since it takes a taller cone to have the same area as a cylinder

with an equal base. But the differences in height are not all that large. Indeed, for most conical cavities of gradual taper, adjusting the semi-empirical parameters in equations 8 or 9 or even in (cylindrical) equation 5 will lead to motors that are just as good. Note also that, although the motor in Example 2 has the poorer fuel, the combination of a smaller nozzle diameter and a larger combustion cavity give it superior performance. The greater fuel density in the Example 1 motor, however, makes it the more reliable of the two.

Conclusion

This theory supplies the necessary know-how for educated amateurs to design Black-Powder rocket motors, whether large or small. And, after building and testing only a few prototypes, it allows them to produce model rockets that perform to their satisfaction.

Here is the procedure: Estimate the intended fuel's power, and choose an initial set of parameters from Table 2. Construct three test motors. Build the first with f_{max} equal to the original guess. Make the other two with f_{max} +10 and f_{max} −10, respectively. Leave all the other parameters, including f_{min}, the same. Test these motors. Since their performance is uncertain, do it under conditions of great prudence and extreme caution; that is, allow for the worst possible failure in any one motor or in all three. See which of them performs best. If that one is satisfactory, the testing phase is complete. If not, test further motors, and focus in, with ever increasing safety and predictability, on ideal performance.

The process is simple and secure. Only the first motors need be an adventure into the unknown. And in the end, anyone can thrill to the lift-off and flight of their own rocket.

Acknowledgment

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References

1) Adapted from E. J. Clinger, *Rockets, Such an Easy Thing!*, Archangelskoye Oblastnoye Aerocosmicheskoye Otdeleniye, Archangelsk, Russia (1990) Part III (in Russian).

2) M. F. Dyunze and V. G. Zhimolokhin, *Solid-Fuel Rocket Motors for Space Systems*, Mashinostroyeniye, Moscow, 1982 (in Russian). The symbols, which come mainly from this source, are sometimes translated and other times transliterated. The resulting notation may be unconventional to rocket experts in either language.

3) We are indebted to A. David Allen and other members of the Ricks College mathematics department for helping us determine this area. (In the conical case, the area should really begin at a height $h > h_{con}$ where the cone becomes tangent to the curve. Equation 6, therefore, represents a slight underestimation of A_{max}. The effects of this approximation, however, are negligible.

4) Dyunze and Zhimolokhin, *op. cit.*, pp 55–57.

5) B. V. Orlov and G. Yu. Mazing, *The Thermodynamic and Ballistic Basis for the Design of Solid-Fuel Rocket Motors*, Third ed., Mashinostroyeniye, Moscow, 1979 p 391 (in Russian).

6) Dyunze and Zhimolokhin, *op. cit.*, pp 68–69.

7) See, for example, *The McGraw-Hill Encyclopedia of Science and Technology*, Seventh ed., McGraw-Hill, New York, Vol. 17 (1992) pp 211–212.